Math 1A: Calculus **Instructor: Alexander Paulin**

Handout: More Differentiation Rules

Discussions 201, 203 // 2018-10-03

1. Sanity checks / Warmup

Problem 1. First, compute the derivative of $1/x$.

Then, compute the derivative of $f(x) = 1/x^2$ in the following ways:

(1) Use the power rule, thinking of $f(x)$ as x^{-2}.
(2) Use the quotient rule, thinking of $f(x)$ as $(1)/(x^2)$.
(3) Use the product rule, thinking of $f(x)$ as $(1/x) \cdot (1/x)$.

Of course, make sure that you get the same answer!

Problem 2. In this problem, we will consider the function $f(x) = e^{2x}$.

Compute the derivative $f'(x)$ in the following ways:

(1) Write $f(x)$ as $(e^2)^x$ and use what you know about derivatives of exponential functions.
(2) Write $f(x)$ as $(e^x) \cdot (e^x)$ and use the product rule.

Check that you get the same answer either way! Once we learn about the chain rule, that will give us yet another way of answering this question.

2. Other problems

Problem 3. Let f, g, h be functions. Express the derivative of the product fgh in terms of f, g, h, f', g', h'. How about the product of four functions?

Problem 4. Use the product rule to show that

$$\frac{d}{dx}(x^2 + 3x + 8)^3 = 3(x^2 + 3x + 8)^2(2x + 3).$$

(The chain rule would be appropriate here too.)

Problem 5. A (magical) box has length 10cm, height 2cm, and width 5cm. If the length is shrinking at a rate of 3cm/s, the height is growing at a rate of 1cm/s, and the width is growing at a rate of 2cm/s, at what rate is the volume of the box changing, in cm^3/s? (Is it increasing or decreasing?)

Problem 6. Use the quotient rule to compute the derivatives of $\cot(x)$, $\sec(x)$, and $\csc(x)$.

Problem 7. Show that

$$\frac{d}{dx}\sin(2x) = 2\cos(2x)$$

in two ways.

(1) Use the definition of the derivative.
(2) Use the angle sum formulas for sine and cosine, together with the product rule for derivatives.

(The chain rule would be appropriate here too.)

Problem 8. Compute

$$\frac{d^{2018}}{dx^{2018}}(e^x \cos(x)).$$

(Hint: compute the first few derivatives and see if you can find a pattern.)